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ABSTRACT

Econometric models built using time series data is usually confronted with the multicollinearity problem. Principal component regression often provides solution, but in cases where the regressors are nonstationary, principal components may yield simple averaging of the regressors. The resulting model is usually difficult to interpret. Sparse principal component analysis can induce the sparsity needed for the components to reflect the relative importance of each regressor. Sparse principal components regression is proposed and applied to an endogenous growth model to illustrate the econometric applications and interpretations.

I. Introduction

The problem of multicollinearity exists when the regressors in a linear model exhibit strong correlations among each other and simultaneously affect least squares estimation of the coefficients. Ordinary least squares estimates will have inflated variance and cases of reverse signs among the estimated coefficients can be expected. One solution in modeling is to drop some variables that may possibly duplicate the effect of other variables. This may however create a void in a modeling framework where the dropped variable may serve as an indicator of a specific component of the framework. Other solutions like principal components regression, ridge regression, and other shrinkage -type of estimators were also proposed in the literature.

Principal components analysis provides a useful tool in modeling since it can create fewer orthogonal aggregates of the variables. In cases of non-stationary time series data however, there is a tendency for the first component to produce simple averaging of all variables. Regression on a component with equal weights for all standardized indicators can result to assignment of similar coefficients to all independent variables. Thus, the relative importance of the indicators in predicting the dependent variable will be masked.

Macroeconomic indicators usually exhibit nonstationary behavior as a consequence of various policies/programs intended to effect movements in the level of these indicators.

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Many econometric models suffer due to the naturally multicollinear predictors. It is possible to observe regression coefficients whose signs are reverse of what is expected from theory.

We propose to extract principal components of the independent variables with the imposition of sparsity-inducing constraint (Zou, et. al., 2006). The sparse components are then used as regressors in a model. The interpretability of the resulting estimates from a sparse principal component regression is illustrated in an endogenous growth model.

II. Endogenous Growth

The evolution of the endogenous growth model started with the pioneering works of (Romer, 1986) and (Lucas, 1988). The long-run growth is fueled by knowledge accumulated by forward-looking, maximizing agents (Romer, 1986). Knowledge is an input in production and has increasing marginal productivity with increases in capital stock. There is a competitive equilibrium assumption with endogenous technological change. Furthermore, growth rates can be increasing over time and large countries can always grow faster than small countries, growth may not even take place to start with. On the other hand, (Lucas, 1988), following the framework of the neoclassical theory of growth and international trade, analyzed the main features of economic development. Physical capital accumulation and technological change are important elements of production. Growth is stimulated only by forces internal to the economy, perhaps dominated by human capital accumulation. Human capital enhances the productivity of both labor and physical capital. The long-run growth rate of economies that are initially poor will be the same as that of the initially wealthy, thus the poor remains to be poor while the rich becomes permanently rich economies. Uniform rates of growth across countries would maintain a perfectly stable distribution of income and wealth over time.

(Pack, 1994) noted that the endogenous growth theory has led to very little tested empirical knowledge. Empirical work should focus on the testing for the implications of the theory more directly, i.e., understanding the economic evolution of the individual countries through the analysis time series data. At the national level, the timing of growth in GDP, investment in machinery, research and development, changes in government macro policy, etc. can be considered.

Growth and interdependence was further analyzed by (Ventura, 1997) using a model that features a technology that exhibits diminishing returns. Countries' ability to trade and eliminate price differentials implies that these diminishing returns are global (affected by

world averages) but not local (unaffected by small countries' actions). The model presents a novel picture of the growth process, requiring a reinterpretation of the source of the conditional convergence.

Sourcing drivers from technology transfer or research and development, countries converge to parallel growth paths, but other countries not following the path stagnate (Howitt, 2000). Variation in per capita income across countries is attributed not only on differences in capital stocks but also because of differences in productivity. (Howitt, 2000) further noted that endogenous growth theory implies the possibility of sustained differences in both levels and rates of growth of national income.

III. Dimensionality, Time Series and Multicollinearity

The vacuum in empirical investigation relating to the endogenous growth theory lies on the analysis at the national level using time series data. In order to minimize the possibility of missing out some important drivers of growth, more indicators or proximate indicators are postulated in the model. These high dimensional set of predictors are also collected/monitored over time. Hence, the problem of high dimensional time series data arises easily.

In time series data of measurements of indicators that benefit from macroeconomic policies and national programs, natural drifting of the variables is expected, resulting to nonstationary behavior. Nonstationarity can easily cause the predictors in a model to exhibit non-orthogonality. Drifting of the time series towards similar trend in a set nonstationary time series can easily yield non-orthogonal character to this set. In a linear model, non-orthogonality of the predictors causes the multicollinearity problem that generally results to instability of the least squares estimates of the regression coefficients. In many cases, the unstable estimates of the regression coefficients can lead to inverse signs relative to the theoretical expectations. One solution to this issue is to drop those duplicating variables, but doing so may void the dynamics being assessed according to some econometric framework.

Principal component regression has been proposed as a possible solution to the problem of multicollinearity. A subset of the orthogonal transformation of the independent variables can be used but with a discrepancy in the amount of information used between the raw individual predictors and the components. Principal component regression may work even if there are more predictors than the number of observations. However, non-stationarity in time series and bulkiness of the number of determinants may result to

uniformity of component loadings and hence can make interpretations difficult. Similarity of loadings in the component can imply that the individual determinants will have contributions to the dependent variable with similar magnitude. Thus, we may fail to understand the relative importance of each determinant in the model.

Focusing on the variance inflation problem caused by multicollinearity, shrinkage estimators are also considered as alternative solutions. Constraints are added in the least squares objective function to produce non-singular design matrix, alleviating variance inflation. The gain in precision is necessarily compensated by the propagation of bias in the parameter estimates. This can also complicate the interpretation of the relative contribution of the individual determinants towards the dependent variable.

Existence of high dimensional data requires tools to visualize it in lower dimensions to facilitate statistical modeling. Principal component regression and many of its variants has been proposed. The common feature added is usually the inclusion of constraints. (Jolliffe, 1982) however cautioned about the misconception that principal components with small eigenvalues will be rarely be of any use in regression. It was demonstrated that these components can be as important as those with large variance, thus, the search of the better way to aggregate high dimensional data or non-orthogonal predictors continues.

A model with infinitely many parameters was proposed by (Goldenshluger and Tsybakov, 2001). To deal with this overparameterized model, an application of blockwise Stein's rule with "weakly" geometrically increasing blocks to the penalized least squares to fit the first N coefficients. Instead of conditional distribution of Y on X , (Heland, 1992) considered the joint distribution of Y and X then work on the fixed number of components. Instead of the usual maximum likelihood estimation for generalized linear regression, in the presence of ill-conditioned design matrix, (Marx and Smith, 1990) proposed an asymptotically biased principal component parameter estimation technique. The principal component regression for generalized linear regression is not always the best choice for model building, but depending on model orientation/specification, it can yield desirable variance properties with minimal bias.

IV. Sparse Principal Component Regression

In modeling involving several non-orthogonal predictors, principal component regression is commonly used. However, if the predictors involved are collected over time, then there is a very high chance that it exhibits non-stationarity, can often result to averaging

of the individual determinants in extracted components. This will result to masking of relative importance of some variables over the others. One econometric solution to this problem is to compute growth rate (differencing) of the indicators instead of the original levels in modeling. Differencing however, may result to an alteration of the dependence structure. Differencing generally filters low data frequencies and preserves the high frequencies, eliminating the effect of some important random shocks, possibly contaminating the econometric relationship being investigated. We are proposing an optimization constraint in the choice components that will be used in the model, hoping to address non-stationarity, non-orthogonality, and high dimensionality of the data.

(Tibshirani, 1996), minimized the residual sum of squares in a model subject to the sum of the absolute value of the coefficients being less than a constant. The constant is also known as the 'lasso' or elastic net. Some coefficients in the model can be forced by the constraints to be exactly 0, facilitating interpretability of the models. In the simulations, the lasso was exhibited to produce some of the favorable properties of both the subset selection (dropping of some variables) and stability of ridge regression (allowing bias in return to variance reduction). By extending the soft thresholding and lasso methods to generalized linear models, (Klinger, 2001) used penalized likelihood estimators for a large number of coefficients. The extension leads to an adaptive selection of model terms without substantial variance inflation.

The principal components analysis intends to summarize many indicators on the same theme into few components in a form of linear combinations. Oftentimes, first component is block (average of all indicators), the rest are contrasts, more difficult to interpret. (Vines, 2000) proposed an algorithm that will produce approximate principal components through 'simplicity preserving' linear transformations. (Rousson and Gasser, 2004) included a constraint in principal component extraction, resulting to suboptimal than ordinary principal components but with simpler, more interpretable components, making the choice of dimensions to be interpreted easier.

(Chipman and Gu, 2005) imposed two constraints in principal component extraction, first (homogeneity constraint) coefficients are constrained to equal a small number of values, second a sparsity constraint. The resultant interpretable directions are either calculated to be close to the original principal component direction or calculated in a stepwise manner that may make the components more orthogonal.

(Zou, et. al., 2006) used lasso as a constraint to principal components extraction, thereby formulated the extraction as a regression problem, resulting in components with sparse loadings. The optimization problem (or sparse principal component analysis criterion) is given by

$$(\hat{\mathbf{a}}, \hat{\mathbf{g}}) = \arg \min_{\mathbf{a}, \mathbf{g}} \sum_{i=1}^n |X_i - \mathbf{a} \mathbf{g}^T X_i|^2 + \mathbf{I} \sum_{j=1}^k |\mathbf{g}_j|^2 + \sum_{j=1}^k \mathbf{I}_{1,j} |\mathbf{g}_j| \quad \text{subject to } \mathbf{a}^T \mathbf{a} = I_k \quad (1)$$

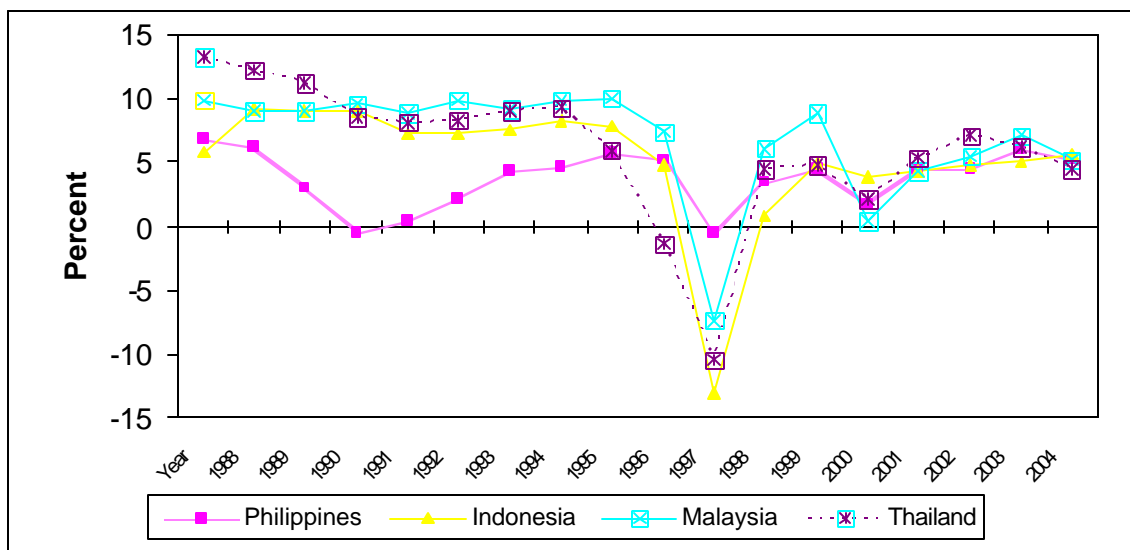
where X_i are the data matrices for each observation i , \mathbf{I} and $\mathbf{I}_{1,j}$ penalizing constants chosen to facilitate existence of solution and convergence of the computational algorithm. The ordinary principal components will solve the optimization problem if $\mathbf{g} = \mathbf{a}$. We follow this approach of imposing the constraints and assess the consequence if the resulting component (sparse) is used in fitting a regression model.

Given the model $Y = X\mathbf{b} + \mathbf{e}$, the ordinary least squares objective function is $\min \|Y - X\mathbf{b}\|^2$ and the corresponding estimate is $\hat{\mathbf{b}} = (X^T X)^{-1} X^T Y$ with variance $V(\hat{\mathbf{b}}) = (X^T X)^{-1} \mathbf{s}^2$. Instead of using all the indicators X , sparse principal components are extracted, then the sparse linear combinations are used in the model, the objective function in least squares estimation is $\min \|Y - \hat{\mathbf{a}} \hat{\mathbf{g}}^T X \mathbf{b}\|^2$. The estimator of the parameters is a solution of the optimization problem in (1), is given by $\hat{\mathbf{b}} = (X^T \hat{\mathbf{g}} \hat{\mathbf{a}}^T \hat{\mathbf{a}} \hat{\mathbf{g}}^T X)^{-1} X^T \hat{\mathbf{g}} \hat{\mathbf{a}}^T Y = (X^T \hat{\mathbf{g}} \hat{\mathbf{g}}^T X)^{-1} X^T \hat{\mathbf{g}} \hat{\mathbf{a}}^T Y$. The variance of the estimator is $V(\hat{\mathbf{b}}) = (X^T \hat{\mathbf{g}} \hat{\mathbf{g}}^T X)^{-1} \mathbf{s}^2$. Note that if instead, principal components are extracted and all components are used in modeling ($\mathbf{g} = \mathbf{a}$), the variance-covariance matrix becomes diagonal, and the magnitude of parameter estimates are the same as in when all the variables are entered directly into the model. When only the subset of the components is used, principal component regression can produce estimates of regression coefficients that are not differentiated from each other, causing complications in interpretations. In sparse principal component regression however, because of the sparsity of the component loadings, the regression coefficients can still differentiate relative contributions of the individual predictors. The orthogonality of the principal components and sparse principal components will also facilitate the computation of the standard error of the parameter estimates.

V. Applications in Endogenous Growth Models

Using data for Philippines, Indonesia, Malaysia, and Thailand from 1988 to 2004, real GDP growth was regressed on the following determinants: X_1 = logarithm of GDP; X_2 =population growth; X_3 =percent share of agriculture to total GDP; X_4 =percent change in food prices; X_5 =percentage of total government expenditure to GDP; X_6 =percent growth in export; X_7 =percent growth in import; X_8 =external debt as a proportion of GNI; and X_9 =debt servicing as a percentage of exports. The growth rates from 1998 to 2004 of the four countries are plotted in Figure 1. The Philippines exhibited a different growth pattern from the other three countries prior to the 1997 Asian financial crisis. While the Philippines had the least reaction to the crisis, the growth patterns of all four countries exhibit similar behavior in the recovery period from the crisis.

Figure 1: GDP Growth Rates of Philippines, Indonesia, Malaysia and Thailand



The augmented Dickey-Fuller (ADF) unit root test yield almost all independent variables nonstationary at 5% level. The possible multicollinearity problem postulated earlier is confirmed, with condition numbers ranging from 135,649 to as much as 1,269,738, all indicative that the problem can seriously ruin the stability of the least squares estimates of the regression coefficients.

Three components were extracted from the predictors for each country. In ordinary principal components extraction, the percentage of the total variance explained by the three components ranges from 77% to 87%. Using sparse principal component extraction criterion, the variance explained by three components ranges from 70% to 79%. The gain in sparsity of the components is paid off by the slightly lower proportion of variance explained by the sparse principal components. The component loading based on standardized determinants are summarized in Appendix 1. Because of the averaging effect of nonstationarity of the determinants, the principal components are difficult to interpret. From the sparse component loadings however, it is easy to interpret that the first component represents the effect of foreign trade on local prices of food in Indonesia, and the structure of the macro-economy for Malaysia, Thailand and Philippines.

Philippines

The ordinary regression on all predictors yield an R^2 of 89%, much lower in principal component regression at 54% and 41% in sparse principal component regression. The estimates of the regression coefficients are given in Table 1. The ordinary least squares of directly regressing growth of GDP to all nine determinants, only the logarithm of GDP yield a significant coefficient. In the principal component regression (PCR), population change (-), percent change in food prices (-), percent of government expenditures to GDP(-), growth in export(+), growth in import(+), and proportion of external debt to GNI (-) yield smaller standard errors. Most of the signs are consistent with theory but a few like percent of government expenditures to GDP seems to exhibit reverse signs.

From the coefficients of sparse principal component regression (SPCR), percent change in food prices (-) and growth in import (+) yield smaller standard errors. Higher food prices hit the local consumers while importation (of food) can possibly facilitate growth of the Philippine economy where the endogenous growth driver may have focused on consumer demand. Consumer spending tends to fuel growth, hence, prices of food commodities and importation can help mitigate the market in favor of the consumers.

Table 1: Regression Coefficients for Philippines

Variable	SPCR		PCR		OLS		
	Coefficient	StdError	Coefficient	StdError	Coefficient	StdErr	p-value
Intercept	13.1164	11.8298	29.4288	10.9058	-139.7940	71.0266	0.0897
X ₁	-0.6548	0.8608	0.2747	0.5119	17.9528	7.4445	0.0467
X ₂	-0.7095	0.9327	-4.3514	1.6043	-8.2364	8.0844	0.3422
X ₃	-0.0381	0.0501	-0.0364	0.0382	-1.1937	0.5542	0.0682
X ₄	-0.1247	0.0405	-0.1414	0.0525	0.1489	0.1369	0.3131
X ₅	.	.	-0.7851	0.2146	-0.4295	0.4955	0.4148
X ₆	0.0030	0.0113	0.0226	0.0145	0.0729	0.0578	0.2473
X ₇	0.1366	0.0492	0.0743	0.0195	0.1417	0.0757	0.1034
X ₈	-0.0030	0.0109	-0.0583	0.0212	0.1078	0.0730	0.1832
X ₉	-0.0546	0.0667	0.0307	0.0380	-0.0174	0.1592	0.9159

Malaysia

The coefficient of determination for ordinary regression on all predictors is 87%, much lower in principal component regression at 61% and 57% in sparse principal component regression. The estimates of the regression coefficients are given in Table 2. Only the external debt as a proportion to GNI yield significant coefficient in the direct regression of growth in GDP to all nine indicators. For the PCR, population change (+), percent change in food prices (-), growth in import (+), and external debt as a proportion to GDP(-) have lower standard errors. Population growth usually exhibits negative effect on growth, in PCR however, it yields positive sign.

The percent share of agriculture to GDP, percent share of external debt to GNI and percent of debt servicing to exports yield lower standard errors in PCR. The positive effect of share of agriculture to GDP can be explained by the possibly improving contribution of agriculture and not having the industries failing to expand significantly. As external debt increases relative to GNI, the country will be forced to finance the debt rather than fuel growth.

Table 2: Regression Coefficients for Malaysia

Variable	SPCR		PCR		OLS		
	Coefficient	StdError	Coefficient	StdError	Coefficient	StdErr	p-value
Intercept	27.4037	8.1140	5.2604	5.2370	-3.5435	80.1790	0.9660
X ₁	-0.7760	0.6083	-0.4386	0.6480	3.3559	5.6440	0.5708
X ₂	.	.	4.8039	1.3100	1.1398	2.3462	0.6419
X ₃	0.1018	0.0798	0.0118	0.0715	0.4002	1.0336	0.7101
X ₄	-0.1160	0.1579	-0.5189	0.1887	-0.7478	0.6370	0.2788
X ₅	0.1350	0.1838	0.1640	0.1299	-0.6717	0.3679	0.1106
X ₆	0.0129	0.0184	0.0087	0.0250	-0.2161	0.1746	0.2557
X ₇	0.0167	0.0131	0.0906	0.0200	0.1381	0.1132	0.2620
X ₈	-0.3620	0.1049	-0.1945	0.0472	-0.3501	0.1263	0.0276
X ₉	0.0562	0.0396	-0.0152	0.0611	0.2788	0.4100	0.5184

Indonesia

In the ordinary regression on all predictors, principal component regression, and in sparse principal component regression, the coefficient of determination is always high (97%, 92% and 84%, respectively). The estimates of the regression coefficients are given in Table 3. Only one factor (percent change in food prices) yield significant parameter estimate in the ordinary regression. PCR yield more determinants with smaller standard errors of estimates than the SPCR. Population growth rates and percent of government expenditures to GDP were included in PCR, but SPCR included percent share of agriculture to total GDP instead. All the other variables including: percent change in food prices, growth rate of exports, growth rate of imports, and external debt relative to export yield smaller standard errors for both SPCR and PCR, also producing the same signs. Growth of export and import in Indonesia had bigger growth contribution as estimated in SPCR compared to PCR.

Table 3: Regression Coefficients for Indonesia

Variable	SPCR		PCR		OLS		
	Coefficient	StdError	Coefficient	StdError	Coefficient	StdErr	p-value
Intercept	4.8440	7.5642	-6.2624	3.6906	-37.4182	31.6032	0.2751
X ₁	-0.6706	0.5282	0.1992	0.3987	2.6554	2.1286	0.2523
X ₂	0.6204	0.4886	3.0847	0.3952	3.9090	2.6723	0.1869
X ₃	0.1106	0.0871	0.0992	0.0622	-0.2323	0.4400	0.6138
X ₄	-0.0724	0.0096	-0.1062	0.0129	-0.2394	0.0563	0.0038
X ₅	0.3774	0.3917	0.4134	0.1941	-0.0263	0.3583	0.9436
X ₆	0.1393	0.0186	0.0694	0.0154	-0.0744	0.0942	0.4554
X ₇	0.0797	0.0106	0.0488	0.0057	0.0621	0.0514	0.2662
X ₈	-0.0484	0.0064	-0.0585	0.0060	0.0177	0.0384	0.6595
X ₉	0.0410	0.0323	-0.0027	0.0239	0.3692	0.1758	0.0739

Thailand

Relatively higher coefficient of determination is also observed in Thailand with 95% for ordinary regression on all predictors, 87% for principal component regression, and 78% for sparse principal component regression. The estimates of the regression coefficients are given in Table 4. There are more significant determinants estimated from OLS in the model for Thailand, including: growth of export, growth of import, and external debt as a proportion of GNI. There are also fewer parameters with small standard errors in SPCR compared to PCR.

Table 4: Regression Coefficients for Thailand

Variable	SPCR		PCR		OLS		
	Coefficient	StdError	Coefficient	StdError	Coefficient	StdErr	p-value
Intercept	31.8262	6.9061	22.9748	7.7001	45.0701	58.5108	0.4663
X ₁	-1.1848	0.6313	0.8021	0.8725	-1.8135	4.3360	0.6883
X ₂	1.9962	0.6061	0.6665	0.4766	7.3421	3.4664	0.0719
X ₃	0.2779	0.0844	0.1181	0.0700	-0.7055	0.5610	0.2489
X ₄	0.0312	0.0810	-0.2886	0.0855	-0.0655	0.2370	0.7903
X ₅	-0.3539	0.1075	-1.0504	0.1173	-0.5182	0.6545	0.4546
X ₆	-0.0942	0.0302	-0.0669	0.0205	-0.2358	0.0878	0.0313
X ₇	0.0305	0.0093	0.0970	0.0113	0.2148	0.0604	0.0093
X ₈	-0.1829	0.0396	-0.1377	0.0177	-0.1159	0.0449	0.0364
X ₉	-0.0457	0.1187	-0.1701	0.0756	-0.1720	0.2338	0.4860

VI. Conclusions

In an econometric model with determinants that exhibit nonstationary behavior, the multicollinearity problem can easily affect least squares estimation of the parameters. Aside from parameter estimates that are unstable, it can also yield signs that are reversed of what is expected. Principal component regression can help resolve the multicollinearity problem but it may yield models that are difficult to interpret because the first component usually averages all determinants if nonstationarity dominates the predictors. Sparse principal component regression is proposed. The components are extracted with sparsity-enhancing constraints before it is used as regressors in a model. Sparsity in the components can facilitate the interpretability of the resulting model as illustrated in some endogenous growth model for four ASEAN-member countries.

References

- Chipman, H. and Gu, H., 2005. Interpretable dimension reduction. *Journal of Applied Statistics* 32(9): 969-987.
- Goldenshluger, A. and Tsybakov, A., 2001. Adaptive prediction and estimation in linear regression with infinitely many parameters. *The Annals of Statistics* 29(6): 1601-1619.
- Helland, I., 1992. Maximum likelihood regression on relevant components. *Journal of the Royal Statistical Society Ser. B* 54(2): 637-647.
- Howitt, P., 2000. Endogenous growth and cross-country income difference. *The American Economic Review* 90(4): 829-946.
- Jolliffe, I., 1982. A note on the use of principal components in regression. *Applied Statistics* 31(3): 300-303.
- Klinger, A., 2001. Inference in high dimensional generalized linear models based on soft thresholding. *Journal of the Royal Statistical Society Ser. B* 63(2): 377-392.
- Lucas, R., 1988. On the mechanics of economic development. *Journal of Monetary Economics* 22(1): 3-42.
- Marx, B. and Smith, E., 1990. Principal component estimation for generalized linear regression. *Biometrika* 77:23-31.
- Pack, H., 1994. Endogenous growth theory: Intellectual appeal and empirical shortcomings. *The Journal of Economic Perspectives* 8(1):55-72.
- Romer, P., 1986. Increasing returns and long-run growth. *The Journal of Political Economy* 94(5):1002-1037.
- Rousson, V. and Gasser, T., 2004. Simple component analysis. *Applied Statistics* 53(4): 539-555.
- Tibshirani, R., 1996. Regression shrinkage and selection via the LASSO. *Journal of the Royal Statistical Society Ser. B* 58(1): 267-288.
- Ventura, J., 1997. Growth and interdependence. *The Quarterly Journal of Economics* 112(1):57-84.
- Vines, S., 2000. Simple principal components. *Applied Statistics* 49(4): 441-451.
- Zou, H., Hastie, T., and Tibshirani, R., 2006. Sparse principal component analysis. *Journal of Computational and Graphical Statistics* 15(2): 265-286.

Appendix 1: Loadings of Principal Components and Sparse Principal Components

Country	Variable	Loadings of SPC			Loadings of PC		
		SPC1	SPC2	SPC3	PC1	PC2	PC3
Indonesia	X ₁	0	-0.47885	0	-0.01331	-0.50772	-0.01676
Indonesia	X ₂	0	0.431075	0	0.368977	0.349509	-0.03786
Indonesia	X ₃	0	0.559054	0	0.065038	0.545049	-0.06826
Indonesia	X ₄	-0.46994	0	0	-0.4095	0.167894	0.501166
Indonesia	X ₅	0	0	-1	-0.26896	0.162897	-0.74424
Indonesia	X ₆	0.468516	0	0	0.454584	-0.01462	0.246501
Indonesia	X ₇	0.562715	0	0	0.457652	-0.12847	0.087629
Indonesia	X ₈	-0.49296	0	0	-0.43737	0.107254	0.324975
Indonesia	X ₉	0	0.521862	0	0.10971	0.48994	0.120485
Malaysia	X ₁	-0.47914	0	0	-0.43336	0.230426	-0.1346
Malaysia	X ₂	0	0	0	0.21478	0.280563	-0.55336
Malaysia	X ₃	0.584526	0	0	0.484191	-0.11373	0.182092
Malaysia	X ₄	0	0.547691	0	0.212151	0.458223	0.441127
Malaysia	X ₅	0	-0.82094	0	-0.02304	-0.65404	-0.09144
Malaysia	X ₆	0.319056	0.143262	0	0.366782	0.321661	0.06257
Malaysia	X ₇	0.41592	0	0	0.410367	-0.01591	-0.33881
Malaysia	X ₈	0	0	1	-0.17723	-0.0127	0.522371
Malaysia	X ₉	0.392392	-0.07463	0	0.389547	-0.3371	0.219049
Philippines	X ₁	0.574885	0	0	0.453093	-0.05864	0.049089
Philippines	X ₂	0.414215	0	0	0.385166	-0.15376	0.441013
Philippines	X ₃	0.553056	0	0	0.445888	0.135822	0.154308
Philippines	X ₄	0.210102	0.109767	0.247695	0.30937	0.327198	0.443377
Philippines	X ₅	0	0	0	-0.23057	-0.18703	0.435428
Philippines	X ₆	0.222073	-0.224	0	0.310635	-0.39131	-0.08869
Philippines	X ₇	0.258089	0	-0.96884	0.328222	0.13449	-0.54423
Philippines	X ₈	-0.17886	0.181415	0	-0.28796	0.402784	0.271997
Philippines	X ₉	0	0.951244	0	0.107325	0.692089	-0.12278
Thailand	X ₁	-0.31439	0.231765	0	-0.34054	0.424593	-0.28151
Thailand	X ₂	0.46225	0	0	0.411762	0.078009	0.148661
Thailand	X ₃	0.48984	0	0	0.421521	-0.16259	0.153601
Thailand	X ₄	0	0.48101	0	0.206134	0.522163	0.367531
Thailand	X ₅	-0.46343	0	0	-0.40793	-0.01004	0.307746
Thailand	X ₆	0.253101	0	0.377781	0.321917	0.156391	0.368065
Thailand	X ₇	0.351775	0	0	0.357461	-0.1481	-0.2237
Thailand	X ₈	-0.21205	0	0.925895	-0.29181	0.003379	0.642233
Thailand	X ₉	0	-0.84553	0	-0.10649	-0.68414	0.217469