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ABSTRACT

This paper introduces new methods of estimating Value-at-Risk (VaR) using Range-Based GARCH (General Autoregressive Conditional Heteroskedasticity) models. These models, which could be either based on the Parkinson Range or Garman-Klasss Range, are applied to 10 stock market indices of selected countries in the Asia-Pacific Region. The results are compared using the traditional methods such as the econometric method based on the ARMA-GARCH models and RiskMetrics™. The performance of the different models is assessed using the out-of-sample VaR forecasts. Series of likelihood ratio (LR) tests namely: LR of unconditional coverage (LR_{uc}), LR of independence (LR_{ind}), and LR of conditional coverage (LR_{cc}) are performed for comparison. The result of the assessment shows that the model based on the Parkinson Range GARCH (1,1) with Student's t distribution is the best performing model on the 10 stock market indices. It has a failure rate, defined as the percentage of actual return that is smaller than the one-step-ahead VaR forecast, of zero in 9 out of 10 stock market indices. The finding of this paper is that Range-Based GARCH Models are good alternatives in modeling volatility and in estimating VaR.

I. Introduction

The need to manage risk has been highlighted in the 1990's by the large losses reported by some financial institutions (Jorion, 2000). For example, in February 1993, Japan's Showa Shell Sekiyu oil company lost \$1.58 billions from speculating on exchange rates (Holton, 2003). In December 1994, California's Orange County announced its losses totaling \$1.8 billions from repos and other transactions (Jorion, 2000). And in February 1995, Nick Leeson, a trader from Britain's Barings PLC, lost \$1.33 billions from unauthorized Nikkei futures trading (Jorion, 2000).

The examples above and other publicized losses in the 1990's have demonstrated the need to control risk. In order to control risk, there should be a way on how to measure it. Measuring risk is tricky because it is not observable, but financial analysts have found a way to quantify it. One method in quantifying risk is the Value-at-Risk (VaR). It is the most popular method and has been adopted by financial institutions like the JP Morgan and Goldman Sachs, and regulators like the Basel Committee on Banking Supervision.

The Basel Committee on Banking Supervision (or Basel Committee) is an international body formed to formulate recommendations on how to regulate banks. Its recent recommendation released in 2004, called the Basel II Accord, is a move to control risk by requiring the banks to hold capital proportional to risks. One of the risks included in the Basel II accord is the market risk which is estimated based on the VaR framework¹. The exact methodology of estimating VaR is flexible. It could be based on the standardized procedure proposed by the Basel Committee or based on the banks proprietary VaR measure as approved by the regulators of the implementing country (i.e., central banks). Aside from managing risk, VaR is needed in the banking sector to comply with the regulatory requirement of the Basel II Accord.

The Value-at-Risk (VaR) is defined as the amount the market value of an asset (or a portfolio of assets) could decline over a certain period under normal market conditions at a specified probability (Tsay, 2005). More formally (following Bao, Lee and Saltoglu (2006)), let r_1, r_2, \dots, r_T be the financial return series and suppose that $\{r_t\}$ follows a stationary stochastic process,

$$r_t = \mathbf{m}_t + \mathbf{e}_t = \mathbf{m}_t + h_t^{1/2} u_t \quad (1)$$

where $\mathbf{m}_t = E[r_t | I_{t-1}]$ is the conditional mean, $h_t = E[\mathbf{e}_t^2 | I_{t-1}]$ is the conditional variance and $u_t = \frac{\mathbf{e}_t}{h_t^{1/2}}$ has a conditional distribution function $F(u_t)$.

The VaR with a given tail probability $\alpha \in (0,1)$, denoted by VaR_α , is defined as the conditional quantile,

$$\Phi(VaR_\alpha) = \mathbf{a} \quad (2)$$

The VaR_α can be estimated by inverting the distribution function, $\Phi(\cdot)$,

$$VaR_\alpha = \Phi^{-1}(\mathbf{a}) = \mathbf{m}_t + h_t^{1/2} F^{-1}(\mathbf{a}) \quad (3)$$

In estimating VaR, we need to specify \mathbf{m}_t, h_t and $F(u_t)$.

There are many methods that can be used to estimate the VaR. The popular methods are the RiskMetrics™ and econometric procedures based on the Autoregressive Moving Average (ARMA) models to specify μ_t and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to specify h_t .

This paper proposes new methods in estimating VaR using the range of the prices of assets. Two Range-based models, the Parkinson Range (using the highest and lowest prices) and the Garman-Klass Range (using the highest, lowest, opening and closing prices), are used to estimate the time varying volatility ($h_t^{1/2}$) needed in estimating VaR.

The remainder of the paper is organized as follows: section 2 discusses the ARMA-GARCH and the RiskMetrics™ approaches in estimating VaR. The ranged-based models are introduced in section 3 while section 4 discusses the methods of assessing the VaR forecasts, using series of Likelihood Ratio tests. Section 5 presents the results of the empirical exercise and section 6 concludes.

RiskMetrics™ and Econometric Approaches to VaR Estimation RiskMetrics

A method in calculating VaR that it is widely used by practitioners is the RiskMetrics™ (Giot and Laurent, 2003). The method was developed by JP Morgan (Morgan and Reuters, 1996) and the VaR is defined as,

$$VaR_{\mathbf{a}} = \mathbf{m} + h_t^{1/2} F^{-1}(\mathbf{a}) \quad (4)$$

where $F(\cdot)$ is the standard normal distribution (example: $F^{-1}(0.01)=2.326$ and $F^{-1}(0.05)=1.645$), $\mu = 0$ and the conditional variance h_t is defined as an Integrated GARCH (IGARCH) with fixed parameters given by,

$$h_t = 0.94h_{t-1} + 0.06r_{t-1}^2 \quad (5)$$

RiskMetrics™ can be easily implemented in a spreadsheet program since the values of the parameters are fixed.

ARMA-GARCH Models

The ARMA-GARCH model is one of the existing methods to estimate VaR. This approach utilizes two models: one for the conditional mean specification (μ_t) and the other for the conditional variance specification (h_t) of the return error series. The mean equation can be defined from the class of models under the AutoRegressive Moving Average (example:ARMA(1,1)), while the variance specification, usually follows the generalized AutoRegressive Conditional Heteroskedasticity (GARCH (1,1)) model (Bollerslev, 1986). A typical ARMA (1,1)-GARCH(1,1) model is defined as,

$$\begin{aligned} r_t &= c + \mathbf{f}_1 r_{t-1} + \mathbf{q}_1 \mathbf{e}_{t-1} + \mathbf{e}_t \quad \text{where} \quad \mathbf{e}_t \sim WN(0, h_t) \\ h_t &= \mathbf{a}_0 + \mathbf{b}_1 h_{t-1} + \mathbf{a}_1 \mathbf{e}_{t-1}^2 \end{aligned} \quad (6)$$

Other specifications of the variance equation (h_t) were later developed to capture leverage effect of the past error terms. Threshold GARCH (TARCH) process was developed to capture the quadratic leverage effect (Glosten, Jagannathan and Runkle, 1993). Nelson (1991), on the other hand, developed the Exponential GARCH process to capture the exponential leverage effect.

Range-Based Models for VaR Estimation

An alternative method of estimating volatility is to use the Range-Based GARCH model. It is similar to GARCH model for the conditional variance but makes use of the daily opening, closing, high and low values of assets which are readily available. These intra-daily prices are used to compute the daily volatility of returns directly. The GARCH model is then applied to the range to estimate the time-varying conditional variance (h_t). Mapa (2003) made use of the Range-Based GARCH model to forecast volatility of the daily Peso-Dollar exchange rates and showed that the Range-Based GARCH models performed better than their GARCH counterparts using inter-daily returns.

Following Mapa (2003), the Range-Based GARCH model is specified as:

$$\mathbf{m}_t = \mathbf{w} + \sum_{j=1}^q \mathbf{a}_j R_{t-j} + \sum_{i=1}^p \mathbf{b}_i \mathbf{m}_{t-i} \quad (7)$$

where $R_t = \boldsymbol{\mu}_t \mathbf{e}_t$ and $\mathbf{e}_t | I_{t-1} \sim iid(1, \boldsymbol{\Sigma}_t)$.

The time varying parameter μ_t is the conditional standard deviation which is modeled directly from the proxy volatility of an asset R_t .

There are two types of proxy volatility, R_b , which will enter into the Range-Based GARCH model: the Parkinson Range (Parkinson;1980) and the Garman-Klass Range (Garman and Klass; 1980). The Parkinson Range of an asset is defined as,

$$R_{Pt} = \sqrt{\frac{(\log(H_t) - \log(L_t))^2}{4 \log(2)}}, \quad (8)$$

where H_t and L_t denote, respectively, the highest and the lowest prices on day t .

The Garman-Klass Range is an extension of Parkinson Range where the information about opening, p_{t-1} , and closing, p_t , prices are incorporated as follows:

$$R_{GKt} = \sqrt{0.5 \left(\log \frac{H_t}{L_t} \right)^2 - 0.39 \left(\log \frac{p_t}{p_{t-1}} \right)^2}. \quad (9)$$

Mapa (2003) showed that the parameters of the Range-Based GARCH models from equation (7) can be estimated using the quasi-maximum likelihood estimation (QMLE) procedure which produces consistent estimators that are asymptotically distributed as normal.

Assessing the VaR Forecast – Likelihood Ratio Tests

The different models to estimate VaR can be assessed, through backtesting, by comparing the forecasted VaR with the actual loss on a portfolio. If the forecasted VaR is smaller than the actual loss, this phenomenon is termed as a VaR violation. The Basel Committee, as contained in the Basel II Accord, has developed a guideline in interpreting the number of violations given 250 observations or approximately one year of daily data. If one computes for a 99%VaR and the number of violations is 4 or below, the model is in the “green light” zone and incurs no penalty. If the violations are 5 to 9, the model is in the “yellow” zone, but if the violations are 10 or more (roughly 3.6% failure rate) the model is in

the “red” zone. If the model is in the “yellow” or “red” zone, the financial institution would incur a penalty.

Under Bangko Sentral ng Pilipinas (BSP) circular 360, “each bank must meet, on a daily basis, a *capital risk charge* expressed as the higher of (i) last trading day’s VaR number or (ii) an average of the daily VaR measures on each of the preceding 60 trading days multiplied by a multiplication factor. The *multiplication factor* shall be set by the BSP on the basis of its assessment of the quality of the bank’s risk management system subject to an absolute minimum of $k = 3$. Banks will be required to add to this factor a “plus” directly related to the ex-post performance of the model (to be determined on a quarterly basis), thereby introducing a built-in positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on the number of backtesting exceptions (i.e., the number of times that actual/hypothetical loss exceeds the VaR measure) for the past 250 trading days of the reference quarter.”²

The risk charge (RC) for day t is given by,

$$RC_t = \text{Max}[VaR_{t-1}, k \frac{1}{60} \sum_{i=1}^{60} VaR_{t-i}] \quad (10)$$

Depending on the number of VaR violations or exceptions for 250 days, Table 1 below provides the penalty scheme, ranging from 0 to 1, that will be added to the minimum of 3 that sums up to k in equation (10). If the method used in estimating the daily VaR produces a large number of VaR violations, the bank incurs a larger risk charge since k in equation (10) increases to a maximum of 4 (under the “red zone”).

Table 1. Penalty Scheme for the number of VaR violations in a 99%VaR

Zone	No. of VaR exceptions/ violations in 250 trading days	“Plus” factor
Green zone	0	0.00
	1	0.00
	2	0.00
	3	0.00
	4	0.00
Yellow zone	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red zone	10 or more	1.00

Likelihood Ratio (LR) Tests

The model can also be assessed using a series of Likelihood Ratio (LR) tests if the VaR violation exceeds zero (Christoffersen, 1998). There are three LR tests available in assessing the performance of the different models: (1) LR of unconditional coverage (LR_{uc}), (2) LR of independence (LR_{ind}), and (3) LR of conditional coverage (LR_{cc}). These tests provide us with information related to the possible mis-specification of the models used in estimating the VaR.

Likelihood Ratio test for Unconditional Coverage (LR_{uc})

The LR test statistic for the unconditional coverage is used to test if the proportion of VaR violations (also known as the empirical failure rate) is equal to the pre-specified level α (equal to 1% for a 99% VaR). Mathematically the empirical failure rate, \hat{p}_1 , can be estimated by,

$$\hat{p}_1 = \frac{1}{T} \sum_{t=1}^T I(r_t < VaR_t(\mathbf{a})) = \frac{T_1}{T} \quad (11)$$

where T is the total number of out-of-sample observations and $I(.)$ is the indicator variable which is equal to one if there is VaR violation and zero otherwise and T_1 is the number of VaR violations.

The empirical failure rate is then tested if it is equal to the pre-specified level, $H_0 : p_1 = a$ against the alternative hypotheses, $H_1 : p_1 > a$.

The decision rule whether to reject or accept the null hypothesis, H_0 , is based on LR_{uc} test statistic (Jorion, 2000) and is given by,

$$LR_{uc} = 2 * \log \left[\frac{\left(1 - \frac{T_1}{T}\right)^{T-T_1} * \left(\frac{T_1}{T}\right)^{T_1}}{(1-a)^{T-T_1} * a^{T_1}} \right] \quad (12)$$

It can be shown that LR_{uc} is asymptotically distributed as chi-square with 1 degree of freedom.

A model that rejects the H_0 of the LR_{uc} is considered as an inferior model since the empirical failure rate p_1 is greater than the pre-specified VaR level a . However, accepting H_0 does not necessarily mean that the model is correctly specified since it is possible for the failure rate to be within the pre-specified level a but the series of VaR violations are not independent of each other. This phenomenon is known as clustered VaR violations (e.g. the 4 VaR violations for 250 trading days (green zone) may happened in just one week). According to Christoffersen and Pelletier (2003), a model with clustered VaR violations is indicative of a mis-specified model b. Likelihood Ratio test of Independence (LR_{ind})

If the null hypothesis in the LR_{uc} test ($H_0 : p_1 = a$) is not rejected, the model is then assessed using a second test known as the LR test of independence (LR_{ind}). The test will tell us whether the proportion of the clustered VaR violations is equal to proportion of the independent VaR violations.

Let T_{ij} be defined as the number of days in which state j occurred in one day while state i occurred in the previous day. Thus, T_{00} is the number days without VaR exception that is preceded by a day without VaR exception, T_{10} is the number of days without VaR exception that is preceded by a day with VaR violation, T_{11} is the number of consecutive 2 days with

VaR violations and T_{01} is the number of days with VaR violation that is preceded by day without a VaR violation.

Define the following,

$$\mathbf{p}_0 = \frac{T_{01}}{T_{01} + T_{00}}, \quad \mathbf{p}_1 = \frac{T_{11}}{T_{11} + T_{10}}, \quad \mathbf{p} = \frac{T_{01} + T_{11}}{T_{01} + T_{00} + T_{10} + T_{11}} \quad (13)$$

Here, π_0 is equal to the proportion of VaR violations preceded by non-VaR violation and π_1 is equal to the proportion of two consecutive VaR violations. In the LR_{ind} test we are interested in the hypothesis $H_0 : \mathbf{p}_0 = \mathbf{p}_1$ against the alternative hypothesis $H_1 : \mathbf{p}_0 \neq \mathbf{p}_1$.

The test statistic for the LR_{ind} test is due to Christoffersen (1998) and is defined in Jorion (2001) as,

$$LR_{ind} = 2 * \log \left[\frac{(1 - \hat{\mathbf{p}}_0)^{T_{00}} \hat{\mathbf{p}}_0^{T_{01}} (1 - \hat{\mathbf{p}}_1)^{T_{10}} \hat{\mathbf{p}}_1^{T_{11}}}{(1 - \hat{\mathbf{p}})^{T_{00} + T_{10}} \hat{\mathbf{p}}^{T_{01} + T_{11}}} \right]. \quad (14)$$

The LR_{ind} is asymptotically distributed as chi-square with 1 degree of freedom.

A model that rejects the H_0 of the LR_{ind} test indicates that the VaR violations are not independent (or are clustering). Clustering of VaR violations is a matter of great concern since this can lead to problems for the banks (or any financial institution). On the other hand, a model that accepts the H_0 in the LR_{ind} test needs to be tested again to determine if the model is correctly specified, since it is possible that the proportion of the independent violations (π_0) or clustered VaR violations (π_1) is higher than the pre-specified failure rate, a .

Likelihood Ratio test of Conditional Coverage (LR_{cc})

Assuming that the VaR violations are independent, the third test to be performed is the LR test of conditional coverage, LR_{cc} . The LR_{cc} test has the null hypothesis, $H_0 : \mathbf{p}_0 = \mathbf{p}_1 = \mathbf{a}$, which states that given the VaR violations are independent, $\mathbf{p}_0 = \mathbf{p}_1$, they are equal to the pre-specified failure rate, a . The alternative hypothesis is

that at least one of the π_s is not equal to α . If the null hypothesis of LR_{cc} test is not rejected, it is indicative that the model is correctly specified. The test statistic for the LR_{cc} is the sum of the test statistics for the LR_{uc} and LR_{ind} and is distributed asymptotically as chi-square with 2 degrees of freedom.

$$LR_{cc} = LR_{uc} + LR_{ind} \quad (15)$$

II. Results And Discussion

This study used 10 stock market indices in the Asia-Pacific Region: Australia, China, Hong Kong, Indonesia, Japan, Korea, Malaysia, the Philippines, Singapore and Taiwan. The data consist of daily observations from July 2, 1997 to March 18, 2005. The number of actual observations varies among the stock market indices because of the differences in the number of trading holidays. The number of observations is in the vicinity of 1,900 observations for each country. The first 80% observations or from July 2, 1997 to September 2, 2003 is used for model estimation while the remaining 20% observations (September 3, 2003 to March 18, 2005) is used for out-of-sample forecast evaluation. The models used to compare VaR forecasts are the RiskMetrics, several ARMA-GARCH type of models and the Range-Based GARCH models.

Among the selected models, the best model is the Parkinson-GARCH(1,1) with Student's t distribution since it is able to forecast correctly all the losses (i.e., no VaR violation) in 9 out of 10 stock indices (see Table 2). The second best model is AR(1)-TARCH(2,1) with Student's t distribution followed by the Garman-Klass-GARCH(1,1) with Student's t distribution.

Table 2. Summary of the VaR violations of ten stock market indices

	Number of Observations	AR(1)-ARCH(1), Normal Distribution		AR(1)-TARCH(2,1), Student's <i>t</i> Distribution		Park - GARCH(1,1) Student's <i>t</i> Distribution *		GK- GARCH(1,1) Student's <i>t</i> Distribution *		RiskMetrics TM	
		Violatio	Rate (%)	Violatio	Rate (%)	Violatio	Rate (%)	Violatio	Rate (%)	Violatio	Rate (%)
Australia	394	0	0.00	0	0.00	0	0.00	0	0.00	51	12.94
China	371	1	0.27	0	0.00	0	0.00	0	0.00	128	34.50
Hong Kong	383	2	0.52	0	0.00	0	0.00	0	0.00	1	0.26
Indonesia	372	2	0.54	0	0.00	0	0.00	1	0.27	1	0.27
Japan	376	4	1.06	0	0.00	0	0.00	0	0.00	7	1.86
Korea	380	1	0.26	2	0.53	0	0.00	0	0.00	3	0.79
Malaysia	378	0	0.00	0	0.00	0	0.00	0	0.00	28	7.41
Philippines	383	1	0.26	0	0.00	0	0.00	1	0.26	6	1.57
Singapore	390	0	0.00	0	0.00	0	0.00	0	0.00	22	5.64
Taiwan	382	3	0.79	1	0.26	1	0.26	1	0.26	7	1.83

*Using fixed degrees of freedom equal to 5 as suggested by Tsay (2001).

On the other hand, the worst performing VaR methodology is the RiskMetricsTM where the forecasts in the ten stock market indices have VaR violations. Using the Basel II definition, the bank will incur a penalty charge using RiskMetricsTM on stock indices in

Australia, China, Malaysia, and Singapore because the failure rates are in the “red” zone (e.g., 3.6% or greater). If the bank is using the selected ARMA-GARCH and Range-Based GARCH models it will not incur a penalty charge since all VaR violations, if any, are within the “green” zone.

In stock indices where there is at least 1 VaR violation, the models were subjected to the series of likelihood ratio tests. The results of the LR tests are summarized in Table 3 below. The selected ARMA-GARCH and Range-Based GARCH models passed the three LR tests (i.e., accepted the null hypothesis). The results of the LR tests suggest that the number of VaR violations of the ARMA-GARCH and Range-Based GARCH models are within the specified failure rate of 1% (using the LR_{uc} test), moreover the resulting violations do not exhibit clustered violations or are independent of each other (based on LR_{ind} test) and finally, the VaR violations (clustered and non-clustered) are within the specified failure rate $\alpha = 0.01$ (LR_{cc} test).

Table 3. Summary of Likelihood Ratio tests of ten stock market indices*

	AR(1)-ARCH(1), Normal Distribution			AR(1)- TARCH(2,1), Student's Distribution			Park- GARCH(1,1) Student's Distribution (fixed df)			GK-GARCH(1,1) RiskMetrics™ Student's Distribution (fixed df)					
	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}
Australia	-	-	-	-	-	-	-	-	-	-	-	-	R	na	na
China	A	A	A	-	-	-	-	-	-	-	-	-	R	na	na
Hong Kong	A	A	A	-	-	-	-	-	-	-	-	-	A	A	A
Indonesi a	A	A	A	-	-	-	-	-	-	A	A	A	A	A	A
Japan	A	A	A	-	-	-	-	-	-	-	-	-	A	A	A
Korea	A	A	A	A	A	A	-	-	-	-	-	-	A	A	A
Malaysia	-	-	-	-	-	-	-	-	-	-	-	-	R	na	na

	AR(1)-ARCH(1), Normal Distribution			AR(1)- TARCH(2,1), Student's Distribution			Park- GARCH(1,1) Student's Distribution (fixed df)			GK-GARCH(1,1) RiskMetrics™ Student's Distribution (fixed df)					
	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}	LR _{uc}	LR _{in} d	LR _{cc}
Philippines	A	A	A	-	-	-	-	-	-	A	A	A	A	A	A
Singapore	-	-	-	-	-	-	-	-	-	-	-	-	R	na	na
Taiwan	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A

*Legend: A = accept the null hypothesis; R = reject the null hypothesis; na = test not applicable; - = cell indicates the test is undefined because the VaR violation is zero.

** Likelihood Ratio (LR) tests: LR_{uc} = LR test of unconditional coverage, LR_{ind} = LR test of independence, LR_{cc} = LR test of conditional coverage. All LR tests are based on 95% confidence interval.

In the case of RiskMetrics™, however, the VaR violations in some stock indices are too high that the null hypothesis of the LR test of unconditional coverage (empirical failure rate is 0.01) is rejected. The performance of RiskMetrics™ will produce higher capital charges in the four indices: Australia, China, Malaysia, and Singapore

Regardless of the specification of the model (GARCH, EGARCH, TARCH), econometric models based on the Student's t distribution tend to forecast VaR correctly (i.e., zero VaR violations) as shown in Table 4. This result is consistent with the findings of Mapa (2003) that the Student's t tend give a better forecast than normal distribution. The reason is that VaR forecast are usually larger because Student's t distribution has fatter tails than the normal distribution.

Table 4. Number of times there is zero VaR violation in 10 stock market indices.

ARMA-GARCH model	Normal Distribution	Student's t Distribution
AR (1)-ARCH (1)	3 out of 10	8 out of 10
AR (1)-GARCH (1,1)	1 out of 10	7 out of 10
AR (1)-GARCH (2,1)	1 out of 10	7 out of 10
AR (1)-GARCH (1,2)	1 out of 10	7 out of 10
AR (1)-GARCH (2,2)	1 out of 10	7 out of 10
AR (1)-TARCH (1,1)	0 out of 10	7 out of 10
AR (1)-TARCH (2,1)	0 out of 10	8 out of 10
AR (1)-TARCH (1,2)	0 out of 10	7 out of 10
AR (1)-TARCH (2,2)	0 out of 10	7 out of 10
AR (1)-EGARCH (1,1)	0 out of 10	6 out of 10
AR (1)-EGARCH (2,1)	0 out of 10	6 out of 10
AR (1)-EGARCH (1,2)	0 out of 10	6 out of 10
AR (1)-EGARCH (2,2)	0 out of 10	6 out of 10
PARKINSON-GARCH (1,1)	0 out of 10	9 out of 10
GARMAN-KLASS-GARCH (1,1)	0 out of 10	7 out of 10

III. Conclusions

This paper introduces a relatively simple yet efficient way of modeling volatility needed in estimating VaR using the Range-Based GARCH models. Two Range-based models were introduced, the Parkinson Range-GARCH and the Garman-Klass GARCH models. The empirical analysis, using 10 stock market indices in the Asia Pacific region, showed that these models are promising based on their out-of-sample performance. In particular, the Parkinson Range-GARCH model was able to produce VaR estimates with zero violation in 9 out of 10 stock market indices. This paper has shown that indeed Range-Based GARCH models are good alternative in modeling volatility and estimating VaR.

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¹ Duffie and Singleton (2003) argues that VaR "captures only one aspect of market risk and is too narrowly defined to be used on its own as a sufficient measure of capital adequacy." Moreover, Artzner et al (1999) showed that VaR does not satisfy the sub-additive property of the risk measure resulting to a serious limitations when aggregating risk.

² Bangko Sentral ng Pilipinas (BSP) Circular No. 360 Annex A.